Doxastic Modal Logic and AGM

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The so called "**AGM** model" for belief revision formulated by Carlos Alchourron, Peter Gärdenfors, and David Makinson in their seminal paper [1] is now widely acknowledged as a standard framework for representing a belief dynamics. This model reconstructs three possible ways of a theory change by considering doxastic (epistemic) actions, the operation of "expansion" among them. Expansion consists in adding some new information to the given belief set and is usually defined through a simple set-theoretic union [2]. Each doxastic action is subject to certain postulates needed for a characterization of their essential features.

By taking the idea of representing the doxastic actions as a kind of modal operators (see [4]), we propose *doxastic modal logic* (**DML**) as a suitable new tool for investigating a theory change. In **DML** expansion is treated as a kind of modal operator: $+A$ is to be understood as "A is added to a given theory". The **AGM** postulates for expansion can be adequately expressed within **DML** and proven as logical theorems.

Closure. After expanding any belief set, the resulting set must be closed under logical implication: if $\mathbb{K}$ is a belief set, then $\mathbb{K} + A$ is also a belief set. This postulate expresses the principle of categorial correspondence [3]. In **DML** this postulate can be expressed by the following axiom scheme analogues to scheme K of normal modal logic:

$$+(A \to B) \to (+A \to +B).$$

Success. If we expand a theory by proposition A, it must belong to the resulting theory: $A \in \mathbb{K} + A$. In **DML** the postulate of success is expressed by the following scheme analogues to the modal axiom T:

$$+A \to A.$$

Inclusion. If we add proposition A to a theory, the resulting theory must include the initial theory as a subset: $\mathbb{K} + A \supseteq \mathbb{K}$. In **DML** the postulate of inclusion can be expressed as follows:

$$B \wedge +A \to B.$$

Vacuity. If we try to expand a theory with a proposition which already is included in it, the theory is not changed. That is, if $a \in K$, then $K + a = K$. In **DML** we have:

$$A \to (+A \to A).$$

Monotonicity. The operation of expansion is monotone with respect to including. If $K \subseteq H$, then $K + a \subseteq H + a$. Analogously the operator of expansion in **DML** is monotone with respect to implication.

$$(B \to C) \to ((B \wedge +A) \to (C \wedge +A)).$$

Minimality. For any belief set $\mathbb{K}$ and proposition A, $\mathbb{K} + A$ is the smallest belief set, which satisfies the postulates of closure, success and inclusion. Extending a theory by any proposition A, we must add to the belief state only A and the sentences resulting from a deductive closure of thus extended theory. That is, if B does not follow from A, then expansion of theory by A must not entail expansion of theory by B.

$$\sim(A \to B) \to \sim(+A \to +B).$$

It is also possible to reconstruct and prove in **DML** the postulates for contraction by using the interconnections between contraction and expansion. Thus, the interpretation of doxastic actions as modal operators allow us to formulate an axiomatic logical system in which the **AGM** postulates are fully representable. This system exposes new properties of doxastic actions, which are in accord with the traditional interpretation.

Литература

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